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Zero-mass representations of Poincaré supersymmetries with arbitrary spinor generators

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Abstract. All possible graded extensions of the Poincaré Lie algebra are constructed. Massless $P^2=0$ representations of these algebras are investigated and it is found that the usual conditions on massless particle helicity states necessarily imply constraints on the spinorial generators. It is shown that these constraints are not consistent with the graded Lie algebra, except in the case of conventional supersymmetry. It follows that massless representations of such algebras are forbidden.

1. Introduction

One of the more exciting developments of the supersymmetry concept has been the introduction of supergravity by Freedman *et al* (1976) and Deser and Zumino (1976). More recently the pure supergravity theory has been extended to include matter couplings, with a special kind of coupling being described by the so-called O(n)-extended supergravity. The largest of these models (if there are no helicity states greater than two) has an SO(8) internal symmetry and provides a unified gauge theory for describing all interactions; strong, weak, electromagnetic and gravitational. This is a true unified model, for not only does it contain all interactions but all particles lie in one supermultiplet. Nevertheless, as Gell-Mann (1977) has pointed out, the model does have phenomenological difficulties. The internal symmetry group SO(8) does not contain the desired minimal gauge group SU(3) × SU(2) × U(1) required to describe the strong, weak and electromagnetic sectors, and missing from the particle supermultiplet are several particles (e.g. muon and its neutrino) which are known to exist experimentally. This suggests that the model is not quite large enough, and instead we should perhaps consider O(9) or O(10) models, although this would mean removing the restriction that the highest helicity is two. There are no conclusive arguments for or against these higher spin fields and recently there have been investigations into field theories with spin $\frac{5}{2}$ (Berends *et al* 1979), pure supergravity with spin 2 and spin $\frac{5}{2}$ (Tchrakian 1979), and massless supersymmetric Lagrangians with arbitrary spin (Curtright 1979). However, a requirement that any spin-$\frac{5}{2}$ fields are gauge fields would imply a symmetry or supersymmetry with spin-$\frac{3}{2}$ generators, and the existence of such symmetries in a massive theory is strictly forbidden by the theorem of Haag *et al* (1975). No complete or satisfactory discussion of the zero mass situation was attempted by these authors. In this paper we demonstrate that massless theories with such higher spin supersymmetries are also forbidden. We do this by first constructing all possible graded extensions of the Poincaré algebra and then show that if the massless $P^2=0$ representations of such algebras satisfy all the usual massless particle helicity conditions, the fermionic generators must satisfy a set of constraints. We show that, apart from the case where the fermionic generators are spin-$\frac{1}{2}$ operators, the constraints are not consistent with the supersymmetry algebra and so deduce that massless representations are forbidden. This in turn prevents the construction of a conventional gauge theory with the gauge fields belonging to massless representations.

2. Graded extensions of the Poincaré algebra

Our analysis of the possible graded extensions of the Poincaré algebra follows closely the work of Konopel'chenko (1977). We start by expressing all the generators in two-component spinor notation with dotted and undotted indices (see for example Zumino 1974). The operator $S^{(j,k)}_{a_1\ldots a_{2j}\dot b_1\ldots\dot b_{2k}}$ then transforms as the $D(j, k)$ representation of the Lorentz group SL(2, $\mathbb{C}$).

The generators of Lorentz transformations, $J_{\mu\nu}$, are replaced by two symmetric spin tensors J_{ab} and $\bar J_{\dot a\dot b}$ which transform according to the representations $D(1,0)$ and $D(0,1)$ respectively and the translation generator P_μ is replaced by $P_{a\dot b}$ (representation $D(\frac{1}{2},\frac{1}{2})$). The connection between the tensor notation and the dotted and undotted indices is then given by

$$
\begin{aligned}
&J_{\mu\nu}=\tfrac{1}{4}[(\sigma_\mu)_{a\dot c}(\sigma_\nu)_{b\dot c}J_{ab}+(\sigma_\mu)_{a\dot c}(\sigma_\nu)_{a\dot d}\bar J_{\dot c\dot d}],\\
&J_{ab}=(\sigma_\mu)_{a\dot e}(\sigma_\nu)_{b\dot e}(J_{\mu\nu}/2),\qquad \bar J_{\dot c\dot d}=(\sigma_\mu)_{a\dot c}(\sigma_\nu)_{a\dot d}(J_{\mu\nu}/2),\\
&P_\mu=(\sigma_\mu)_{a\dot b}(P_{a\dot b}/2),\qquad P_{a\dot b}=(\sigma_\mu)_{a\dot b}P_\mu,
\end{aligned}
\tag{1}
$$

where $\sigma_\mu=(\mathbf{1},\sigma_i)$ are the Pauli matrices†.

If the original generators are represented by Hermitian operators, then $\bar J_{\dot a\dot b}=J^\dagger_{ab}$ and $P^\dagger_{a\dot b}=P_{a\dot b}$ (where † denotes Hermitian conjugation).

The Poincaré algebra can now be written as

$$
\begin{aligned}
&[J_{ab},J_{cd}]=\mathrm{i}(\epsilon_{ac}J_{bd}+\epsilon_{bc}J_{ad}+\epsilon_{ad}J_{cb}+\epsilon_{bd}J_{ca}),\\
&[\bar J_{\dot a\dot b},\bar J_{\dot c\dot d}]=\mathrm{i}(\epsilon_{\dot a\dot c}\bar J_{\dot b\dot d}+\epsilon_{\dot b\dot c}\bar J_{\dot a\dot d}+\epsilon_{\dot a\dot d}\bar J_{\dot c\dot b}+\epsilon_{\dot b\dot d}\bar J_{\dot c\dot a}),\\
&[J_{ab},\bar J_{\dot c\dot d}]=0,\qquad [J_{ab},P_{c\dot d}]=\mathrm{i}(\epsilon_{ac}P_{b\dot d}+\epsilon_{bc}P_{a\dot d}),\\
&[\bar J_{\dot a\dot b},P_{c\dot d}]=\mathrm{i}(\epsilon_{\dot a\dot d}P_{c\dot b}+\epsilon_{\dot b\dot d}P_{c\dot a}),\qquad [P_{a\dot b},P_{c\dot d}]=0.
\end{aligned}
\tag{2}
$$

This algebra is then extended to a graded Lie algebra by adding 'odd' generators $Q^{(j,k)}_{a_1\ldots a_{2j}\dot b_1\ldots\dot b_{2k}}$ which transform according to some representation of the Poincaré group. We have chosen $Q^{(j,k)}$ to transform under Lorentz transformations according to the representation $D(j, k)$ and at this stage we have not defined the commutator bracket $[P_{a\dot b}, Q^{(j,k)}]$. To preserve the correct spin statistics the $Q^{(j,k)}$ must be spinors or spinor-tensors, that is $j+k$ must be half-integral (Streater and Wightman 1964).

Under Lorentz transformations $Q^{(j,k)}$ transforms as

$$
\begin{aligned}
&[J_{ab},Q^{(j,k)}_{a_1\ldots a_{2j}\dot b_1\ldots\dot b_{2k}}]=\mathrm{i}\operatorname*{Sym}_{a_1\ldots a_{2j}}\epsilon_{aa_1}Q^{(j,k)}_{ba_2\ldots a_{2j}\dot b_1\ldots\dot b_{2k}},\\
&[\bar J_{\dot a\dot b},Q^{(j,k)}_{a_1\ldots a_{2j}\dot b_1\ldots\dot b_{2k}}]=\mathrm{i}\operatorname*{Sym}_{\dot b_1\ldots\dot b_{2k}}\epsilon_{\dot a\dot b_1}Q^{(j,k)}_{a_1\ldots a_{2j}\dot b\dot b_2\ldots\dot b_{2k}},
\end{aligned}
\tag{3}
$$

† Summation over repeated spin indices (dotted or undotted) is defined as $X_aY_a=X_a\epsilon_{ab}Y_b\equiv X_1Y_2-X_2Y_1$ and $X_{\dot a}Y_{\dot a}=X_{\dot a}\epsilon_{\dot a\dot b}Y_{\dot b}\equiv X_{\dot 1}Y_{\dot 2}-X_{\dot 2}Y_{\dot 1}$.

where $\mathrm{Sym}_{a\ldots}$ denotes symmetrisation with respect to the corresponding set of indices. To close the algebra we have to define the commutator $[P_{a\dot{b}}, Q^{(j,k)}]$ and anticommutator $\{Q^{(j,k)}, Q^{(j,k)}\}$. Possible candidates for the closure of these (anti)commutators can be found by an examination of the decomposition of the two-tensor products $D(\frac{1}{2}, \frac{1}{2}) \times D(j, k)$ and $D(j, k) \times D(j, k)$. We have

$$D(\tfrac{1}{2}, \tfrac{1}{2}) \times D(j, k) = D(j+\tfrac{1}{2}, k+\tfrac{1}{2}) + D(j+\tfrac{1}{2}, k-\tfrac{1}{2}) + D(j-\tfrac{1}{2}, k+\tfrac{1}{2}) + D(j-\tfrac{1}{2}, k-\tfrac{1}{2}), \tag{4a}$$

$$D(j, k) \times D(j, k) = D(2j, 2k) + \ldots + D(0, 0). \tag{4b}$$

As there is no generator in our algebra transforming by any of the representations on the right-hand side of (4*a*) the commutator $[P, Q^{(j,k)}]$ must be zero and from (4*b*) we see that the only possibility for the anticommutator $\{Q^{(j,k)}, Q^{(j,k)}\}$ is for it to be proportional to J_{ab} or $\bar{J}_{\dot{a}\dot{b}}$. However we may use the generalised Jacobi identity (Kac 1977)

$$[A, [B, C]] = [[A, B], C] + (-1)^{\alpha\beta}[B, [A, C]], \tag{5}$$

where [,] is a generalised bracket product such that $[A, B] = (-1)^{\alpha\beta}[B, A]$, and α, β are zero if A, B are even generators and one if A, B are odd generators, to determine the constant of proportionality. A simple substitution shows this to be zero.† Thus there is no extension of the Poincaré algebra by a single spinor generator $Q^{(j,k)}_{a_1\ldots a_{2j}\dot{b}_1\ldots\dot{b}_{2k}}$.

We now investigate algebras containing two spinor generators, $Q^{(j,k)}$ and $\bar{Q}^{(j',k')}$. We follow the same procedure as before but to close the algebra, we now have to define the following commutators and anticommutators: $[P, Q]$, $[P, \bar{Q}]$, $\{Q, Q\}$, $\{\bar{Q}, \bar{Q}\}$ and $\{Q, \bar{Q}\}$. This requires an examination of the following tensor products (besides (4))

$$D(\tfrac{1}{2}, \tfrac{1}{2}) \times D(j', k') = D(j'+\tfrac{1}{2}, k'+\tfrac{1}{2}) + D(j'+\tfrac{1}{2}, k'-\tfrac{1}{2}) + D(j'-\tfrac{1}{2}, k'+\tfrac{1}{2}) + D(j'-\tfrac{1}{2}, k'-\tfrac{1}{2}), \tag{6a}$$

$$D(j, k) + D(j', k') = D(j+j', k+k') + \ldots + D(|j-j'|, |k-k'|). \tag{6b}$$

There are now several possibilities. The first is to take $j - j' = 0$ and $|k - k'| = 1$ (alternatively $|j - j'| = 1$, $k - k' = 0$) leaving us with $[P, Q]$ and $[P, \bar{Q}]$ both zero, and $\{Q, \bar{Q}\}$, $\{Q, Q\}$ and $\{\bar{Q}, \bar{Q}\}$ all proportional to either J_{ab} or $\bar{J}_{\dot{a}\dot{b}}$. As in the case of a single spinor generator $Q^{(j,k)}$ the generalised Jacobi identity (5) may be used to show that all the constants of proportionality are zero.

The other possibility open to us is to take $|j - j'|$ and $|k - k'| = \frac{1}{2}$, allowing the commutation relations

$$[P, Q] \sim \bar{Q}, \qquad [P, \bar{Q}] \sim Q, \qquad \{Q, \bar{Q}\} \sim P \tag{7a}$$

and

$$\{Q, Q\} \sim J(\bar{J}), \qquad \{\bar{Q}, \bar{Q}\} \sim J(\bar{J}). \tag{7b}$$

We have already indicated that the anticommutation relations in (7*b*) are zero by the generalised Jacobi identity (5). This leaves the (anti)commutators (7*a*) to be defined. Before doing this we note that the conditions $|j - j'| = \frac{1}{2}$, $|k - k'| = \frac{1}{2}$ separate into two classes: (i) the algebra contains spinor generators $Q^{(j,k)}$ and $\bar{Q}^{(j-1/2,k+1/2)}$ and (ii) the algebra contains spinor generators $Q^{(j,k)}$ and $\bar{Q}^{(j+1/2,k+1/2)}$. Furthermore we can restrict the possibilities even further by requiring Hermitian representations. Under

† This is not true for $Q^{(\frac{1}{2},0)}$ or $Q^{(0,\frac{1}{2})}$, but eventually this can be made zero by asking for the theory to be invariant under Hermitian conjugation—so we take the constant to be zero in general.

Hermitian conjugation a representation $D(j, k)$ of the Lorentz group goes over to a representation $D(k, j)$. Thus an algebra can only have an Hermitian representation if it contains both the generators $Q^{(j,k)}$ and $\bar{Q}^{(k,j)}$. It is easy to see that this requirement is only satisfied by generators in class (i), in which case the Poincaré algebra is extended by spinorial generators $Q^{(j,j-1/2)}$ and $\bar{Q}^{(j-1/2,j)}$.

We can now write out the commutation relations (7a) explicitly:

$$[P_{a\dot{b}}, Q^{(j,j-1/2)}_{a_1\ldots a_{2j}\dot{b}_1\ldots\dot{b}_{2j-1}}] = A \underset{a_1\ldots a_{2j}}{\mathrm{Sym}}\ \epsilon_{aa_1}\bar{Q}^{(j-1/2,j)}_{a_2\ldots a_{2j}\dot{b}\dot{b}_1\ldots\dot{b}_{2j-1}}, \tag{8a}$$

$$[P_{a\dot{b}}, \bar{Q}^{(j-1/2,j)}_{a_1\ldots a_{2j-1}\dot{b}_1\ldots\dot{b}_{2j}}] = B \underset{\dot{b}_1\ldots\dot{b}_{2j}}{\mathrm{Sym}}\ \epsilon_{\dot{b}\dot{b}_1}Q^{(j,j-1/2)}_{aa_1\ldots a_{2j-1}\dot{b}_2\ldots\dot{b}_{2j}}, \tag{8b}$$

$$\{Q^{(j,j-1/2)}_{a_1\ldots a_{2j}\dot{b}_1\ldots\dot{b}_{2j-1}}, \bar{Q}^{(j-1/2,j)}_{c_1\ldots c_{2j-1}\dot{d}_1\ldots\dot{d}_{2j}}\}$$
$$= C \underset{a,\dot{b},c,\dot{d}}{\mathrm{Sym}}\ \epsilon_{a_1c_1}\ldots\epsilon_{a_{2j-1}c_{2j-1}}\epsilon_{\dot{b}_1\dot{d}_1}\ldots\epsilon_{\dot{b}_{2j-1}\dot{d}_{2j-1}}P_{a_{2j}\dot{d}_{2j}}. \tag{8c}$$

The values of the constants A, B and C are determined by substitution into the generalised Jacobi identity (5). This yields $AB = 0$, $AC = 0$ and $BC = 0$, so that there are three possibilities: (i) $A \neq 0, B = 0, C = 0$, (ii) $A = 0 = C, B \neq 0$, and (iii) $A = B = 0$, $C \neq 0$. These yield the corresponding algebras:

$$\text{(i)}\quad [P_{a\dot{b}}, Q^{(j,j-1/2)}_{a_1\ldots a_{2j}\dot{b}_1\ldots\dot{b}_{2j-1}}] = A \underset{a_1\ldots a_{2j}}{\mathrm{Sym}}\ \epsilon_{aa_1}\bar{Q}^{(j-1/2,j)}_{a_2\ldots a_{2j}\dot{b}\dot{b}_1\ldots\dot{b}_{2j-1}}, \tag{9a}$$

$$[P, \bar{Q}] = 0, \qquad \{Q, \bar{Q}\} = 0, \qquad \{Q, Q\} = 0, \qquad \{\bar{Q}, \bar{Q}\} = 0, \tag{9b}$$

$$\text{(ii)}\quad [P_{a\dot{b}}, \bar{Q}^{(j-1/2,j)}_{a_1\ldots a_{2j-1}\dot{b}_1\ldots\dot{b}_{2j}}] = B \underset{\dot{b}_1\ldots\dot{b}_{2j}}{\mathrm{Sym}}\ \epsilon_{\dot{b}\dot{b}_1}Q^{(j,j-1/2)}_{aa_1\ldots a_{2j-1}\dot{b}_2\ldots\dot{b}_{2j}}, \tag{10a}$$

$$[P, Q] = 0, \qquad \{Q, \bar{Q}\} = 0, \qquad \{Q, Q\} = 0, \qquad \{\bar{Q}, \bar{Q}\} = 0, \tag{10b}$$

$$\text{(iii)}\quad [P, Q] = 0, \qquad [P, \bar{Q}] = 0, \qquad \{Q, Q\} = \{\bar{Q}, \bar{Q}\} = 0, \tag{11a}$$

$$\{Q^{(j,j-1/2)}_{a_1\ldots a_{2j}\dot{b}_1\ldots\dot{b}_{2j-1}}, \bar{Q}^{(j-1/2,j)}_{c_1\ldots c_{2j-1}\dot{d}_1\ldots\dot{d}_{2j}}\}$$
$$= C \underset{a,\dot{b},c,\dot{d}}{\mathrm{Sym}}\ \epsilon_{a_1c_1}\ldots\epsilon_{a_{2j-1}c_{2j-1}}\epsilon_{\dot{b}_1\dot{d}_1}\ldots\epsilon_{\dot{b}_{2j-1}\dot{d}_{2j-1}}P_{a_{2j}\dot{d}_{2j}}. \tag{11b}$$

We can impose even further restrictions on the choice of algebras by noting that for algebras of type (i) or (ii) P_μ no longer commutes with Q or $\bar{Q}$ so P^2 is not an invariant of the algebra (that is the theory would have a continuous mass spectrum). Furthermore these algebras are not invariant with respect to Hermitian conjugation and so do not have Hermitian representations. Hence, if we eliminate these algebras as not physical, we are left with one possible form of graded extension of the Poincaré algebra. This involves spinorial generators $Q^{(j,j-1/2)}$ and the Hermitian conjugates $\bar{Q}^{(j-1/2,j)}$ satisfying the algebra (11).

3. Massless representations

In all the algebras constructed in § 2 we assumed that P_μ commuted with the spinorial charges Q and $\bar{Q}$ and so P^2 is a Casimir of the algebra. The irreducible representations of the algebra may then be classified according to the eigenvalues of P^2. These fall into four classes: (i) $P^2 > 0$, (ii) $P^2 < 0$, (iii) $P^2 = 0$, $P_\mu \neq 0$ and (iv) $P_\mu = 0$. Here we will study the possible $P^2 = 0$, $P_\mu \neq 0$ representations of the supersymmetry algebras (11). These massless representations are of particular interest for the construction of a conventional gauge version of the algebra with the gauge fields belonging to a massless representation.

We begin our analysis of massless $P^2=0$ irreducible representations of the supersymmetry algebras by assuming that they are completely reducible into $P^2=0$ representations of the Poincaré algebra. These representations ($P^2=0$) of the Poincaré algebra then fall into two classes (Völkel 1977). The first involves an infinite series of one-dimensional representations of the 'little algebra' (in this case the algebra of the Euclidean group E_2), characterised by $\lambda=0, \pm\frac{1}{2}, \pm 1, \ldots$. The corresponding states are interpreted as particles with zero mass and helicity $\lambda=0, \pm\frac{1}{2}, \pm 1, \ldots$. The second class involves infinite-dimensional unitary representations of the algebra, and as yet there has been no physical application of these—they would imply continuous spin spectra. In the reduction of $P^2=0$ supersymmetric representations to $P^2=0$ Poincaré representations we assume only the first class arises.

With these assumptions we know that for $P^2=0$ supersymmetric representations of the Poincaré algebra, the Pauli-Lubanski vector

$$W_\mu = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu J^{\rho\sigma} \tag{12}$$

satisfies

$$P^2 = W^2 = 0$$

and the helicity Λ becomes a Casimir of the Poincaré algebra with the generators satisfying the constraint (Boyce *et al* 1967)

$$W_\mu \equiv \Lambda P_\mu \qquad \text{where} \qquad \Lambda = P_0^{-1}\epsilon_{0\nu\rho\sigma}P^\nu J^{\rho\sigma}. \tag{13}$$

Now consider any tensor operator $T^{(j,k)}_{a_1\ldots a_{2j}\dot b_1\ldots\dot b_{2k}}$ which commutes with P^2 and transforms as the $D(j,k)$ representation of the Lorentz group. Since $[P^2, T^{(j,k)}]=0$, $T^{(j,k)}$ must necessarily transform massless states into massless states for $P^2|m=0\rangle = 0|m=0\rangle=0$, and so

$$P^2T^{(j,k)}|m=0\rangle = T^{(j,k)}P^2|m=0\rangle = 0. \tag{14}$$

This property must be consistent with the constraint (13) in the sense that

$$[W_\mu, T^{(j,k)}] \equiv [\Lambda P_\mu, T^{(j,k)}]. \tag{15}$$

Under Lorentz transformations J_{ab} and $\bar J_{\dot a\dot b}$, $T^{(j,k)}$ transforms as

$$\begin{aligned}[][J_{ab}, T^{(j,k)}_{a_1\ldots a_{2j}\dot b_1\ldots\dot b_{2k}}] &= \mathrm{i} \underset{a_1\ldots a_{2j}}{\mathrm{Sym}}\ \epsilon_{aa_1}T^{(j,k)}_{ba_2\ldots a_{2j}\dot b_1\ldots\dot b_{2k}},\\ [\bar J_{\dot a\dot b}, T^{(j,k)}_{a_1\ldots a_{2j}\dot b_1\ldots\dot b_{2k}}] &= \mathrm{i} \underset{\dot b_1\ldots\dot b_{2k}}{\mathrm{Sym}}\ \epsilon_{\dot a\dot b_1}T^{(j,k)}_{a_1\ldots a_{2j}\dot b\dot b_2\ldots\dot b_{2k}},\end{aligned} \tag{16}$$

whence it follows from the constraints (13) and (15), that the operator $T^{(j,k)}$ when acting on massless states must satisfy

$$\begin{aligned}&\underset{a_1\ldots a_{2j}}{\mathrm{Sym}}\ \bar P^{\dot b a_1}T^{(j,k)}_{a_1\ldots a_{2j}\dot b_1\ldots\dot b_{2k}} \equiv 0,\\ &\underset{\dot b_1\ldots\dot b_{2k}}{\mathrm{Sym}}\ \bar P^{\dot b_1 a}T^{(j,k)}_{a_1\ldots a_{2j}\dot b_1\ldots\dot n_{2k}} \equiv 0.\end{aligned} \tag{17}$$

By the symmetry of the indices $a_1 \ldots a_{2j}$ and $\dot{b}_1 \ldots \dot{b}_{2k}$ this reduces to

$$\begin{aligned} \bar{P}^{\dot{b}a_i} T^{(j,k)}_{a_1\ldots a_i\ldots a_{2j}\dot{b}_1\ldots\dot{b}_{2k}} &\equiv 0, \qquad i = 1, \ldots 2j \\ \bar{P}^{\dot{b}_i a} T^{(j,k)}_{a_1\ldots a_{2j}\dot{b}_1\ldots\dot{b}_i\ldots\dot{b}_{2k}} &\equiv 0, \qquad i = 1, \ldots 2k \end{aligned} \tag{18}$$

and in particular we must have

$$\bar{P}^{a_1\dot{b}} T^{(j,k)}_{a_1\ldots a_{2j}\dot{b}_1\ldots\dot{b}_{2k}} \equiv 0. \tag{19}$$

The identity (19) will have very strong implications when we consider $T^{(j,k)}$ to be the generators of a graded extension of the Poincaré algebra. For instance, if we take $T^{(j,k)}$ to be $Q^{(j,j-1/2)}$ and $\bar{Q}^{(j-1/2,j)}$ (the generators of the 'supersymmetry' algebra (11)), and investigate massless ($P^2 = 0$) representations of the algebra, then $Q^{(j,j-1/2)}$ must satisfy the constraint (19). Moreover this constraint must be consistent with the algebra in the sense that, if we consider the anticommutator (11*b*)

$$\begin{aligned} &\{Q^{(j,j-1/2)}_{a_1\ldots a_{2j}\dot{b}_1\ldots\dot{b}_{2j-1}}, \bar{Q}^{(j-1/2,j)}_{c_1\ldots c_{2j-1}\dot{d}_1\ldots\dot{d}_{2j}}\} \\ &\qquad = C \operatorname*{Sym}_{a,\dot{b},c,\dot{d}} \epsilon_{a_1c_1} \ldots \epsilon_{a_{2j-1}c_{2j-1}} \epsilon_{\dot{b}_1\dot{d}_1} \ldots \epsilon_{\dot{b}_{2j-1}\dot{d}_{2j-1}} P_{a_{2j}\dot{d}_{2j}} \end{aligned} \tag{11b}$$

and apply the operator $\bar{P}^{\dot{b}a_1}$ to both sides, we must consistently get zero when we are restricted to massless states. Clearly this is always true for the left-hand side because of the constraint (19); however, applying $\bar{P}^{\dot{b}a_1}$ to the right-hand side of (11*b*) will only give zero from those terms involving $P_{a_1\dot{d}_i}$ ($i = 1, \ldots 2j$) (because $P^2 = 0$) and it is straightforward to see that the remainder will not in general be identically zero. The only situation where the right-hand side will be identically zero is when there are no ϵ tensors present, in which case $\bar{P}^{\dot{b}a_1}$ will always contract with $P_{a_1\dot{d}_1}$ to give zero by $P^2 = 0$. The algebras with no ϵ tensors present are precisely those with spinor generators $Q^{(1/2,0)}_a$ and $Q^{(0,1/2)}_{\dot{a}}$—the conventional supersymmetry algebras (Fayet and Ferrara 1977). It is the requirement that for higher spin, odd generators $Q^{(j,j-1/2)}$ and $\bar{Q}^{(j-1/2,j)}$, $j > \frac{1}{2}$, the anticommutator $\{Q, \bar{Q}\}$ be constructed from $P_{a\dot{b}}$ together with ϵ tensors that prevents the algebra from being compatible with the usual helicity constraints for massless particle states. We therefore conclude that, apart from conventional supersymmetry, massless representations of graded extensions of the Poincaré algebra are forbidden.

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